

Aufgabe 20

a)

$$\begin{aligned}\sqrt[3]{4} \cdot \sqrt[4]{4} &= 4^{\frac{1}{3}} \cdot 4^{\frac{1}{4}} = 4^{\frac{1}{3} + \frac{1}{4}} = 4^{\frac{4}{12} + \frac{3}{12}} \\ &= 4^{\frac{7}{12}} = \sqrt[12]{4^7} = \sqrt[12]{16.384}\end{aligned}$$

b)

$$\begin{aligned}\sqrt{2} \cdot \sqrt[3]{2} &= 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} = 2^{\frac{1}{2} + \frac{1}{3}} = 2^{\frac{3}{6} + \frac{2}{6}} \\ &= 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}\end{aligned}$$

c)

$$\begin{aligned}\sqrt[5]{3} : \sqrt{3} &= 3^{\frac{1}{5}} : 3^{\frac{1}{2}} = 3^{\frac{1}{5} - \frac{1}{2}} = 3^{\frac{2}{10} - \frac{5}{10}} \\ &= 3^{-\frac{3}{10}} = \frac{1}{3^{\frac{3}{10}}} = \frac{1}{\sqrt[10]{3^3}} = \frac{1}{\sqrt[10]{27}}\end{aligned}$$

d)

$$\begin{aligned}\sqrt[3]{5} : \sqrt{5} &= 5^{\frac{1}{3}} : 5^{\frac{1}{2}} = 5^{\frac{1}{3} - \frac{1}{2}} = 5^{\frac{2}{6} - \frac{3}{6}} \\ &= 5^{-\frac{1}{6}} = \frac{1}{5^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{5}}\end{aligned}$$

e)

$$\begin{aligned}\sqrt[4]{2^9} \cdot \sqrt{2^9} &= 2^{\frac{9}{4}} \cdot 2^{\frac{9}{2}} = 2^{\frac{9}{4} + \frac{9}{2}} = 2^{\frac{9}{4} + \frac{18}{4}} \\ &= 2^{\frac{27}{4}} = \sqrt[4]{2^{27}} = \sqrt[4]{2^4 \cdot 2^4 \cdot 2^4 \cdot 2^4 \cdot 2^4 \cdot 2^3} \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \sqrt[4]{2^3} \\ &= 2^5 \cdot \sqrt[4]{2^3} = 32\sqrt[4]{8}\end{aligned}$$

$$f) \sqrt{\frac{2}{3}} : \sqrt{\frac{2}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} : \left(\frac{2}{3}\right)^{\frac{1}{2}} = \left(\frac{2}{3}\right)^{\frac{1}{2}-\frac{1}{2}} = \left(\frac{2}{3}\right)^0 = 1$$

Alternative: $\sqrt{\frac{2}{3}} : \sqrt{\frac{2}{3}} = 1$

$$g) \sqrt[3]{2x} : \sqrt[3]{x} = \frac{\sqrt[3]{2x}}{\sqrt[3]{x}} = \sqrt[3]{\frac{2x}{x}} = \sqrt[3]{2} \quad \text{für } x \neq 0$$

$$h) \sqrt{x} \cdot \sqrt{4x} = \sqrt{4x \cdot x} = \sqrt{4x^2} = 2 \cdot |x| \quad \text{für } x \in \mathbb{R}$$

$$i) \sqrt[4]{10y} : \sqrt[4]{2y} = \frac{\sqrt[4]{10y}}{\sqrt[4]{2y}} = \sqrt[4]{\frac{10y}{2y}} = \sqrt[4]{10} \quad \text{für } y \neq 0$$

$$j) \sqrt[5]{x^2} \cdot \sqrt[5]{x^3} = \sqrt[5]{x^2 \cdot x^3} = \sqrt[5]{x^5} = x$$

$$k) \sqrt[3]{\frac{1}{x}} \cdot \sqrt[3]{x^2} = \sqrt[3]{\frac{1}{x} \cdot x^2} = \sqrt[3]{\frac{x^2}{x}} = \sqrt[3]{x} \quad \text{für } x \neq 0$$

$$l) \sqrt[n]{3} : \sqrt[n]{\frac{2}{3}} = \sqrt[n]{3 : \frac{2}{3}} = \sqrt[n]{3 \cdot \frac{3}{2}} = \sqrt[n]{\frac{9}{2}} = \sqrt[n]{4\frac{1}{2}}$$