

Aufgabe 4

a)

$$x^4 - 20x^2 + 64 = 0$$

Sei $x^2 = z$

$$z^2 - 20z + 64 = 0$$

$$z_{1,2} = -\frac{-20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 - 64}$$

$$= 10 \pm \sqrt{36}$$

$$z_1 = 10 + 6 = 16$$

$$z_2 = 10 - 6 = 4$$

$$x^2 = 16$$

$$x_{1,2} = \pm 4$$

$$x^2 = 4$$

$$x_{3,4} = \pm 2$$

b)

$$2x^4 - 8x^2 - 90 = 0$$

$$x^4 - 4x^2 - 45 = 0$$

Sei $x^2 = z$

$$z^2 - 4z - 45 = 0$$

$$z_{1,2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 + 45}$$

$$= 2 \pm \sqrt{49}$$

$$z_1 = 2 + 7 = 9$$

$$z_2 = 2 - 7 = -5$$

$$x^2 = 9$$

$$x_{1,2} = \pm 3$$

$$x^2 = -5$$

Keine Lösung in \mathbb{R} .

c)

$$3x^4 + 9x^2 - 162 = 0$$

$$x^4 + 3x^2 - 54 = 0$$

Sei $x^2 = z$

$$z^2 + 3z - 54 = 0$$

$$z_{1,2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 54}$$

$$= -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{216}{4}}$$

$$= -\frac{3}{2} \pm \sqrt{\frac{225}{4}}$$

$$z_1 = -\frac{3}{2} + \frac{15}{2} = \frac{12}{2} = 6$$

$$z_2 = -\frac{3}{2} - \frac{15}{2} = -\frac{18}{2} = -9$$

$$x^2 = 6$$

$$x_{1,2} = \pm\sqrt{6}$$

$$x^2 = -9$$

Keine Lösung in \mathbb{R} .

d)

$$x^4 + \frac{4}{9}x^2 - \frac{13}{9} = 0$$

Sei $x^2 = z$

$$z^2 + \frac{4}{9}z - \frac{13}{9} = 0$$

$$\begin{aligned} z_{1,2} &= -\frac{\frac{4}{9}}{2} \pm \sqrt{\left(\frac{\frac{4}{9}}{2}\right)^2 + \frac{13}{9}} \\ &= -\frac{4}{18} \pm \sqrt{\left(\frac{4}{18}\right)^2 + \frac{13}{9}} \\ &= -\frac{4}{18} \pm \sqrt{\frac{16}{324} + \frac{468}{324}} \\ &= -\frac{4}{18} \pm \sqrt{\frac{484}{324}} \\ z_1 &= -\frac{4}{18} + \frac{22}{18} = \frac{18}{18} = 1 \\ z_2 &= -\frac{4}{18} - \frac{22}{18} = -\frac{26}{18} = -1\frac{4}{9} \end{aligned}$$

$$x^2 = 1$$

$$x_{1,2} = \pm 1$$

$$x^2 = -1\frac{4}{9}$$

Keine Lösung in \mathbb{R} .

f)

$$x^6 - 10x^3 + 9 = 0$$

Sei $x^3 = z$

$$z^2 - 10z + 9 = 0$$

$$\begin{aligned} z_{1,2} &= -\frac{-10}{2} \pm \sqrt{\left(\frac{10}{2}\right)^2 - 9} \\ &= -5 \pm \sqrt{25 - 9} \\ &= -5 \pm \sqrt{16} \\ z_1 &= -5 + 4 = -1 \\ z_2 &= -5 - 4 = -9 \end{aligned}$$

$$x^3 = -1$$

$$x_1 = -1$$

$$x^3 = -9$$

$$x_2 = -3$$