# Aufgabe 3

$$x_0 = 2$$

**a)**

$$f(x) = x^2$$

$$\begin{aligned} m_s &= \frac{(2+h)^2 - (2)^2}{h} \\ &= \frac{4 + 4h + h^2 - 4}{h} \\ &= \frac{4h + h^2}{h} \\ &= 4 + h \end{aligned}$$

$$m_t = \lim_{h \to 0} (4 + h) = 4$$

**b)**

$$f(x) = \frac{2}{x}$$

$$\begin{aligned} m_s &= \frac{\frac{2}{2+h} - \frac{2}{2}}{h} \\ &= \frac{\frac{2}{2+h} - 1}{h} \\ &= \frac{\frac{2}{2+h} - \frac{2+h}{2+h}}{h} \\ &= \frac{\frac{2-(2+h)}{2+h}}{h} \end{aligned}$$

$$= \frac{\frac{2-2-h}{2+h}}{h}$$
$$= \frac{\frac{-h}{2+h}}{h}$$
$$= \frac{-h}{2+h} \cdot \frac{1}{h}$$
$$= \frac{-1}{2+h}$$

$$m_t = \lim_{h \to 0} \left( \frac{-1}{2+h} \right) = -\frac{1}{2}$$

**c)**

$$f(x) = 2x^2 - 3$$

$$m_s = \frac{2 \cdot (2+h)^2 - 3 - (2 \cdot 2^2 - 3)}{h}$$
$$= \frac{2 \cdot (4 + 4h + h^2) - 3 - 5}{h}$$
$$= \frac{8 + 8h + 2h^2 - 8}{h}$$
$$= \frac{8h + 2h^2}{h}$$
$$= 8 + 2h$$

$$m_t = \lim_{h \to 0} (8 + 2h) = 8$$

**d)**

$$f(x) = x^4$$

$$\begin{aligned}
m_s &= \frac{(2+h)^4 - (2)^4}{h} \\
&= \frac{(2+h)^2(2+h)^2 - 16}{h} \\
&= \frac{(4+4h+h^2)(4+4h+h^2) - 16}{h} \\
&= \frac{16 + 16h + 4h^2 + 16h + 16h^2 + 4h^3 + 4h^2 + 4h^3 + h^4 - 16}{h} \\
&= \frac{h^4 + 8h^3 + 24h^2 + 32h}{h} = h^3 + 8h^2 + 24h + 32
\end{aligned}$$

$$m_t = \lim_{h \to 0}(h^3 + 8h^2 + 24h + 32) = 32$$

**e)**

$$f(x) = x^3$$

$$\begin{aligned}
m_s &= \frac{(2+h)^3 - (2)^3}{h} \\
&= \frac{(2+h)(2+h)^2 - 8}{h} \\
&= \frac{(2+h)(4+4h+h^2) - 8}{h} \\
&= \frac{8 + 8h + 2h^2 + 4h + 4h^2 + h^3 - 8}{h} \\
&= \frac{h^3 + 6h^2 + 12h}{h} = h^2 + 6h + 12
\end{aligned}$$

$$m_t = \lim_{h \to 0}(h^2 + 6h + 12) = 12$$

**f)**

$$f(x) = 4x - x^2$$

$$\begin{aligned}
m_s &= \frac{4 \cdot (2+h) - (2+h)^2 - (4 \cdot 2 - 2^2)}{h} \\
&= \frac{8 + 4h - (4 + 4h + h^2) - 4}{h} \\
&= \frac{8 + 4h - 4 - 4h - h^2 - 4}{h} \\
&= \frac{-h^2}{h} = -h
\end{aligned}$$

$$m_t = \lim_{h \to 0}(-h) = 0$$

**g)**

$$f(x) = -\frac{1}{x} = \frac{-1}{x}$$

$$\begin{aligned}
m_s &= \frac{\frac{-1}{2+h} - \left(\frac{-1}{2}\right)}{h} \\
&= \frac{\frac{2 \cdot (-1)}{2 \cdot (2+h)} + \frac{1 \cdot (2+h)}{2 \cdot (2+h)}}{h} \\
&= \frac{\frac{-2+2+h}{4+2h}}{h} \\
&= \frac{h}{4+2h} \cdot \frac{1}{h} \\
&= \frac{1}{4+2h} \\
m_t &= \lim_{h \to 0}\left(\frac{1}{4+2h}\right) = \frac{1}{4}
\end{aligned}$$

**h)**

$$f(x) = 5$$

Das heißt:

$$f(2+h) = 5 \quad \text{und} \quad f(2) = 5$$

$$m_s = \frac{5-5}{h} = 0$$

$$m_t = \lim_{h \to 0}(0) = 0$$