

Aufgabe 5

a)

$$\begin{aligned}x^5 - 20x^3 + 64x &= 0 \\x(x^4 - 20x^2 + 64) &= 0 \\x_1 &= 0 \\x^4 - 20x^2 + 64 &= 0\end{aligned}$$

Sei $x^2 = z$

$$z^2 - 20z + 64 = 0$$

$$\begin{aligned}z_{1,2} &= -\frac{-20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 - 64} \\&= 10 \pm \sqrt{36} \\z_1 &= 10 + 6 = 16 \\z_2 &= 10 - 6 = 4\end{aligned}$$

$$\begin{aligned}x^2 &= 16 \\x_{2,3} &= \pm 4\end{aligned}$$

$$\begin{aligned}x^2 &= 4 \\x_{4,5} &= \pm 2\end{aligned}$$

b)

$$\begin{aligned}x^5 - 17x^3 + 16x &= 0 \\x(x^4 - 17x^2 + 16) &= 0 \\x_1 &= 0 \\x^4 - 17x^2 + 16 &= 0\end{aligned}$$

Sei $x^2 = z$

$$z^2 - 17z + 16 = 0$$

$$\begin{aligned}z_{1,2} &= -\frac{-17}{2} \pm \sqrt{\left(\frac{17}{2}\right)^2 - 16} \\&= \frac{17}{2} \pm \sqrt{\frac{289}{4} - \frac{64}{4}} \\&= \frac{17}{2} \pm \sqrt{\frac{225}{4}} \\z_1 &= \frac{17}{2} + \frac{15}{2} = \frac{32}{2} = 16 \\z_2 &= \frac{17}{2} - \frac{15}{2} = \frac{2}{2} = 1\end{aligned}$$

$$\begin{aligned}x^2 &= 16 \\x_{2,3} &= \pm 4\end{aligned}$$

$$\begin{aligned}x^2 &= 1 \\x_{4,5} &= \pm 1\end{aligned}$$

c)

$$\begin{aligned}x^6 + 3x^4 - 54x^2 &= 0 \\x^2(x^4 + 3x^2 - 54) &= 0 \\x_1 &= 0 \\x^4 + 3x^2 - 54 &= 0\end{aligned}$$

Sei $x^2 = z$

$$z^2 + 3z - 54 = 0$$

$$\begin{aligned}z_{1,2} &= -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 54} \\&= -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{216}{4}} \\&= -\frac{3}{2} \pm \sqrt{\frac{225}{4}} \\z_1 &= -\frac{3}{2} + \frac{15}{2} = \frac{12}{2} = 6 \\z_2 &= -\frac{3}{2} - \frac{15}{2} = -\frac{18}{2} = -9\end{aligned}$$

$$x^2 = 6$$

$$x_{2,3} = \pm\sqrt{6}$$

$$x^2 = -9$$

Keine Lösung in \mathbb{R} .

d)

$$\begin{aligned}
 2x^5 - \frac{13}{3}x^3 + 2x &= 0 \\
 x \left(2x^4 - \frac{13}{3}x^2 + 2 \right) &= 0 \\
 x_1 &= 0 \\
 2x^4 - \frac{13}{3}x^2 + 2 &= 0
 \end{aligned}$$

Sei $x^2 = z$

$$2z^2 - \frac{13}{3}z + 2 = 0$$

$$z^2 - \frac{13}{6}z + 1 = 0$$

$$\begin{aligned}
 z_{1,2} &= -\frac{-\frac{13}{6}}{2} \pm \sqrt{\left(\frac{\frac{13}{6}}{2}\right)^2 - 1} \\
 &= \frac{13}{12} \pm \sqrt{\left(\frac{13}{12}\right)^2 - 1} \\
 &= \frac{13}{12} \pm \sqrt{\frac{169}{144} - \frac{144}{144}} \\
 &= \frac{13}{12} \pm \sqrt{\frac{25}{144}} \\
 z_1 &= \frac{13}{12} + \frac{5}{12} = \frac{18}{12} = \frac{3}{2} = 1\frac{1}{2} \\
 z_2 &= \frac{13}{12} - \frac{5}{12} = \frac{8}{12} = \frac{2}{3}
 \end{aligned}$$

$$x^2 = 1\frac{1}{2}$$

$$x^2 = \frac{2}{3}$$

$$x_{2,3} = \pm \sqrt{1\frac{1}{2}}$$

$$x_{4,5} = \pm \sqrt{\frac{2}{3}}$$

e)

$$\left(x - \frac{2}{3}\right) \left(x^4 - \frac{13}{6}x^2 + 1\right) = 0$$

$$x_1 = \frac{2}{3}$$

$$x^4 - \frac{13}{6}x^2 + 1 = 0$$

Sei $x^2 = z$

$$2z^2 - \frac{13}{3}z + 2 = 0$$

$$z^2 - \frac{13}{6}z + 1 = 0$$

$$z_{1,2} = -\frac{-\frac{13}{6}}{2} \pm \sqrt{\left(\frac{\frac{13}{6}}{2}\right)^2 - 1}$$

$$= \frac{13}{12} \pm \sqrt{\left(\frac{13}{12}\right)^2 - 1}$$

$$= \frac{13}{12} \pm \sqrt{\frac{169}{144} - \frac{144}{144}}$$

$$= \frac{13}{12} \pm \sqrt{\frac{25}{144}}$$

$$z_1 = \frac{13}{12} + \frac{5}{12} = \frac{18}{12} = \frac{3}{2} = 1\frac{1}{2}$$

$$z_2 = \frac{13}{12} - \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$$

$$x^2 = 1\frac{1}{2}$$

$$x_{2,3} = \pm \sqrt{1\frac{1}{2}}$$

$$x^2 = \frac{2}{3}$$

$$x_{4,5} = \pm \sqrt{\frac{2}{3}}$$

f)

$$(x^3 - 8) \left(x^4 - \frac{14}{3}x^2 + 5 \right) = 0$$

$$x^3 - 8 = 0$$

$$x^3 = 8$$

$$x_1 = 2$$

$$x^4 - \frac{14}{3}x^2 + 5 = 0$$

Sei $x^2 = z$

$$z^2 - \frac{14}{3}z + 5 = 0$$

$$z_{1,2} = -\frac{-\frac{14}{3}}{2} \pm \sqrt{\left(\frac{\frac{14}{3}}{2}\right)^2 - 5}$$

$$= \frac{14}{6} \pm \sqrt{\left(\frac{14}{6}\right)^2 - 5}$$

$$= \frac{14}{6} \pm \sqrt{\frac{196}{36} - \frac{180}{36}}$$

$$= \frac{14}{6} \pm \sqrt{\frac{16}{36}}$$

$$z_1 = \frac{14}{6} + \frac{4}{6} = \frac{18}{6} = 3$$

$$z_2 = \frac{14}{6} - \frac{4}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$$

$$x^2 = 3$$

$$x_{2,3} = \pm\sqrt{3}$$

$$x^2 = 1\frac{2}{3}$$

$$x_{4,5} = \pm\sqrt{1\frac{2}{3}}$$