

## Aufgabe 15

a)

$$7x^2 - 3 = 4$$

$$7x^2 = 7$$

$$x^2 = 1$$

$$x_{1,2} = \pm 1$$

b)

$$x^4 + 6x^2 = -9$$

$$x^4 + 6x^2 + 9 = 0$$

Sei  $x^2 = z$

$$z^2 + 6z + 9 = 0$$

$$z_{1,2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - 9}$$

$$= -3$$

$$z = -3$$

$$x^2 = -3$$

Keine Lösung in  $\mathbb{R}$ .

c)

$$\begin{aligned}x^3 - x^2 &= 4x^2 \\x^3 - 5x^2 &= 0 \\x^2(x - 5) &= 0 \\x^2 &= 0 \\x_1 &= 0 \\x - 5 &= 0 \\x_2 &= 5\end{aligned}$$

d)

$$\begin{aligned}5x^2 - x &= x + 2x^2 \\3x^2 - 2x &= 0 \\x(3x - 2) &= 0 \\x_1 &= 0 \\3x - 2 &= 0 \\3x &= 2 \\x_2 &= \frac{2}{3}\end{aligned}$$

e)

$$\begin{aligned}3x - 8 &= 5x^2 - 8 \\3x - 5x^2 &= 0 \\x(3 - 5x) &= 0 \\x_1 &= 0 \\3 - 5x &= 0 \\3 &= 5x \\x_2 &= \frac{3}{5}\end{aligned}$$

f)

$$\begin{aligned}x^3 + 2x^2 &= x^2 - 3x^3 \\4x^3 + x^2 &= 0 \\x^2(4x + 1) &= 0 \\x^2 &= 0 \\x_1 &= 0 \\4x + 1 &= 0 \\4x &= -1 \\x_2 &= -\frac{1}{4}\end{aligned}$$

**Aufgabe 16**

a)

$$\begin{aligned}f(x) &= 3 \\x^3 - 2x + 3 &= 3 \\x^3 - 2x &= 0 \\x(x^2 - 2) &= 0 \\x_1 &= 0 \\x^2 - 2 &= 0 \\x^2 &= 2 \\x_{2,3} &= \pm\sqrt{2}\end{aligned}$$

b)

$$\begin{aligned}f(x) &= 3 \\x^3 + x - 7 &= 3 \\x^3 + x - 10 &= 0\end{aligned}$$

Mit GTR (Alternative: *Polynomdivision*):

$$x = 2$$

c)

$$\begin{aligned}f(x) &= 3 \\x^4 - 6x^2 + 3 &= 3 \\x^4 - 6x^2 &= 0 \\x^2(x^2 - 6) &= 0 \\x^2 &= 0 \\x_1 &= 0 \\x^2 - 6 &= 0 \\x^2 &= 6 \\x_{2,3} &= \pm\sqrt{6}\end{aligned}$$

## Aufgabe 17

Wir nutzen die „Produkt gleich Null“-Regel „rückwärts“.

Eine Nullstelle ( $x = 0$ ):

$$f(x) = x^7$$

Alternative ( $x = 1$ ):

$$\begin{aligned}f(x) &= (x - 1)^7 \\&= x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1\end{aligned}$$

Und so weiter.

Zwei Nullstellen ( $x_1 = 0$  und  $x_2 = 1$ ):

$$f(x) = x^6(x - 1) = x^7 - x^6$$

Drei Nullstellen ( $x_1 = 0$ ,  $x_2 = 1$  und  $x_3 = 2$ ):

$$\begin{aligned} f(x) &= x^5(x-1)(x-2) \\ &= x^7 - 3x^6 + 2x^5 \end{aligned}$$

Vier Nullstellen ( $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$  und  $x_4 = 3$ ):

$$\begin{aligned} f(x) &= x^4(x-1)(x-2)(x-3) \\ &= x^7 - 6x^6 + 11x^5 - 6x^4 \end{aligned}$$

Fünf Nullstellen ( $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$ ,  $x_4 = 3$  und  $x_5 = 4$ ):

$$\begin{aligned} f(x) &= x^3(x-1)(x-2)(x-3)(x-4) \\ &= x^7 - 10x^6 + 35x^5 - 50x^4 + 24x^3 \end{aligned}$$