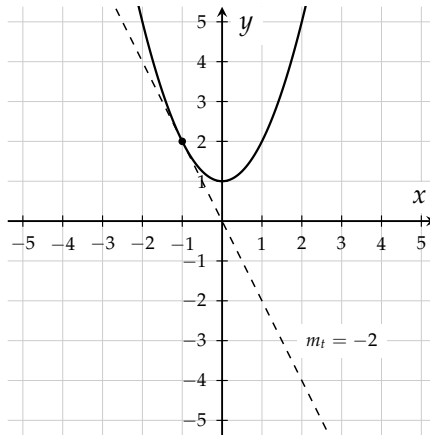


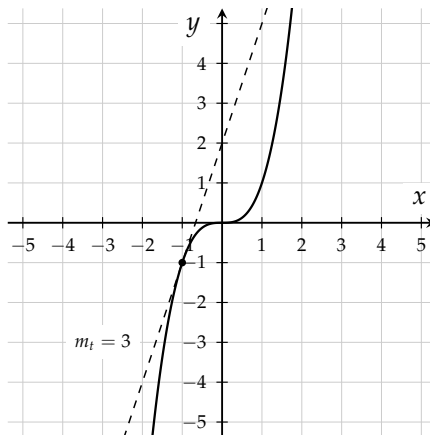
Aufgabe 4

a)

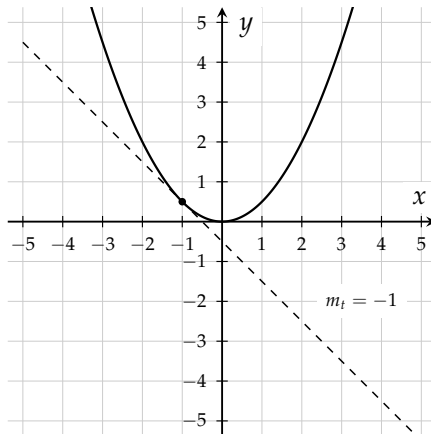
$$f(x) = x^2 + 1$$



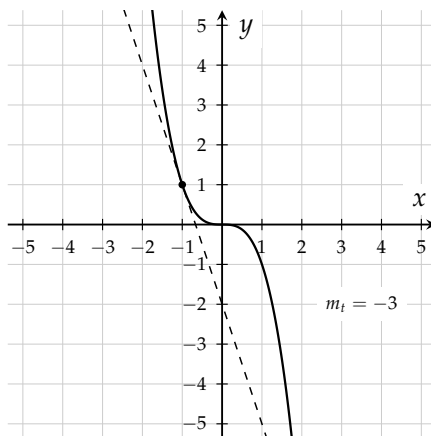
$$f(x) = x^3$$



$$f(x) = 0,5 \cdot x^2$$



$$f(x) = -x^3$$



b)

$$\begin{aligned}f(x) &= x^2 + 1 \\m_s &= \frac{f(-1+h) - f(-1)}{h} \\&= \frac{(-1+h)^2 + 1 - ((-1)^2 + 1)}{h} \\&= \frac{h^2 - 2h + 1 + 1 - 2}{h} \\&= \frac{h^2 - 2h}{h} \\&= h - 2 \\m_t &= \lim_{h \rightarrow 0} (h - 2) = -2\end{aligned}$$

$$\begin{aligned}f(x) &= x^3 \\m_s &= \frac{f(-1+h) - f(-1)}{h} \\&= \frac{(-1+h)^3 - ((-1)^3)}{h} \\&= \frac{(h-1)^2(h-1) + 1}{h} \\&= \frac{(h^2 - 2h + 1)(h-1) + 1}{h} \\&= \frac{h^3 - 2h^2 + h - h^2 + 2h - 1 + 1}{h} \\&= \frac{h^3 - 3h^2 + 3h}{h} \\&= h^2 - 3h + 3 \\m_t &= \lim_{h \rightarrow 0} (h^2 - 3h + 3) = 3\end{aligned}$$

$$\begin{aligned}f(x) &= 0,5 \cdot x^2 \\m_s &= \frac{f(-1+h) - f(-1)}{h} \\&= \frac{0,5 \cdot (-1+h)^2 - 0,5 \cdot (-1)^2}{h} \\&= \frac{0,5 \cdot (h^2 - 2h + 1) - 0,5}{h} \\&= \frac{0,5h^2 - h + 0,5 - 0,5}{h} \\&= \frac{0,5h^2 - h}{h} \\&= 0,5h - 1 \\m_t &= \lim_{h \rightarrow 0} (0,5h - 1) = -1\end{aligned}$$

$$\begin{aligned}f(x) &= -x^3 \\m_s &= \frac{f(-1+h) - f(-1)}{h} \\&= \frac{-(-1+h)^3 - (-(-1)^3)}{h} \\&= \frac{-(h-1)^2(h-1) - 1}{h} \\&= \frac{-(h^2 - 2h + 1)(h-1) - 1}{h} \\&= \frac{-(h^3 - 2h^2 + h - h^2 + 2h - 1) - 1}{h} \\&= \frac{-(h^3 - 3h^2 + 3h - 1) - 1}{h} \\&= \frac{-h^3 + 3h^2 - 3h + 1 - 1}{h}\end{aligned}$$

$$\begin{aligned} &= \frac{-h^3 + 3h^2 - 3h}{h} \\ &= -h^2 + 3h - 3 \\ m_t &= \lim_{h \rightarrow 0} (-h^2 + 3h - 3) = -3 \end{aligned}$$