Aufgabe 1

$$m_t = f'(x)$$

a)

$$t(x) = m_t \cdot x + b \qquad x_0 = 4$$

$$\begin{aligned} f'(x) &= x \\ f'(4) &= 4 = m_t \end{aligned}$$

$$t(x) = 4x + b$$

$$\begin{aligned} f(4) &= 0{,}5 \cdot 4^2 = 8 \\ 8 &= 4 \cdot 4 + b \\ 8 &= 16 + b \\ -8 &= b \end{aligned}$$

$$t(x) = 4x - 8$$

b)

$$t(x) = m_t \cdot x + b \qquad x_0 = 1$$

$$\begin{aligned} f'(x) &= 16x^3 - 2x \\ f'(1) &= 16 - 2 = 14 = m_t \end{aligned}$$

$$t(x) = 14x + b$$

$$\begin{aligned} f(1) &= 4 \cdot 1^4 - 1^2 = 3 \\ 3 &= 14 \cdot 1 + b \\ 3 &= 14 + b \\ -11 &= b \end{aligned}$$

$$t(x) = 14x - 11$$

c)

$$t(x) = m_t \cdot x + b \qquad x_0 = -1$$

$$f'(x) = 1{,}5x^2 - 1$$
$$f'(-1) = 1{,}5 - 1 = 0{,}5 = m_t$$

$$t(x) = 0{,}5x + b$$

$$f(0{,}5) = 0{,}5 \cdot (-1)^3 - (-1) = 0{,}5$$
$$0{,}5 = 0{,}5 \cdot (-1) + b$$
$$0{,}5 = -0{,}5 + b$$
$$1 = b$$

$$t(x) = 0{,}5x + 1$$

d)

$$t(x) = m_t \cdot x + b \qquad x_0 = 0{,}5$$

$$f'(x) = 8x^3 + 12x^2 - 10x$$
$$f'(0{,}5) = 8 \cdot (0{,}5)^3 + 12 \cdot (0{,}5)^2 - 10 \cdot 0{,}5 = 1 + 3 - 5 = -1$$

$$t(x) = -x + b$$

$$f(0{,}5) = 2 \cdot 0{,}5^4 + 4 \cdot 0{,}5^3 - 5 \cdot 0{,}5^2 = -0{,}625$$
$$-0{,}625 = -0{,}5 + b$$
$$-0{,}125 = b$$

$$t(x) = -x - 0{,}125$$

Aufgabe 2

Zwei Geraden verlaufen parallel, wenn ihre Steigungen gleich sind.

$$g(x) = x - 2 = 1 \cdot x - 2 \quad \Rightarrow \quad m = 1$$

Gesucht sind die Stellen, an denen die Tangenten an den Graphen von f die Steigung $m_t = 1$ besitzen.

a)

$$\begin{aligned} f'(x) &= x = m_t \\ x &= 1 \end{aligned}$$

Die Tangente an der Stelle $x = 1$ ist parallel zur Geraden g.

b)

$$\begin{aligned} f'(x) &= -2x \\ -2x &= 1 \\ x &= -\frac{1}{2} \end{aligned}$$

Die Tangente an der Stelle $x = -\frac{1}{2}$ ist parallel zur Geraden g.

c)

$$\begin{aligned} f'(x) &= 3x^2 \\ 3x^2 &= 1 \\ x^2 &= \frac{1}{3} \\ x_{1,2} &= \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} \end{aligned}$$

Die Tangenten an den Stellen $x_1 = \frac{1}{\sqrt{3}}$ und $x_2 = -\frac{1}{\sqrt{3}}$ sind parallel zur Geraden g.

Aufgabe 3

a)

$$t(x) = m_t \cdot x + b \qquad x_0 = 2$$

$$\begin{aligned} f'(x) &= -2x \\ f'(2) &= -4 = m_t \end{aligned}$$

$$t(x) = -4x + b$$

$$\begin{aligned} f(2) &= -(2)^2 = -4 \\ -4 &= -4 \cdot 2 + b \\ -4 &= -8 + b \\ 4 &= b \end{aligned}$$

$$t(x) = -4x + 4$$

$$\begin{aligned} t(x) &= 0 \\ -4x + 4 &= 0 \\ -4x &= -4 \\ x &= 1 \end{aligned}$$

Die Tangente schneidet die x-Achse an der Stelle $x = 1$.

b)

$$t(x) = m_t \cdot x + b \qquad x_0 = -3$$

$$\begin{aligned} f'(x) &= 4x - 1 \\ f'(-3) &= 4 \cdot (-3) - 1 = -13 = m_t \end{aligned}$$

$$t(x) = -13x + b$$

$$\begin{aligned} f(-3) &= 2 \cdot (-3)^2 - (-3) = 21 \\ 21 &= -13 \cdot (-3) + b \\ 21 &= 39 + b \\ -18 &= b \end{aligned}$$

$$t(x) = -13x - 18$$

$$\begin{aligned} t(x) &= 0 \\ -13x - 18 &= 0 \\ -13x &= 18 \\ x &= -\frac{18}{13} = -1\frac{5}{13} \end{aligned}$$

Die Tangente schneidet die x-Achse an der Stelle $x = -1\frac{5}{13}$.

c)

$$t(x) = m_t \cdot x + b \qquad x_0 = 4$$

$$\begin{aligned} f'(x) &= -2x \\ f'(4) &= -2 \cdot 4 = -8 = m_t \end{aligned}$$

$$t(x) = -8x + b$$

$$\begin{aligned} f(4) &= -(4)^2 + 3 = -16 + 3 = -13 \\ -13 &= -8 \cdot 4 + b \\ -13 &= -32 + b \\ 19 &= b \end{aligned}$$

$$t(x) = -8x + 19$$

$$
\begin{aligned}
t(x) &= 0 \\
-8x + 19 &= 0 \\
19 &= 8x \\
x &= \frac{19}{8} = 2\frac{3}{8}
\end{aligned}
$$

Die Tangente schneidet die x-Achse an der Stelle $x = 2\frac{3}{8}$.

d)

$$t(x) = m_t \cdot x + b \qquad x_0 = 0$$

$$
\begin{aligned}
f'(x) &= -5 \\
f'(0) &= -5 = m_t
\end{aligned}
$$

$$t(x) = -5x + b$$

$$
\begin{aligned}
f(0) &= -5 \cdot 0 + 3 = 3 \\
3 &= -5 \cdot 0 + b \\
3 &= b
\end{aligned}
$$

$$t(x) = -5x + 3$$

$$
\begin{aligned}
t(x) &= 0 \\
-5x + 3 &= 0 \\
-5x &= -3 \\
x &= \frac{3}{5}
\end{aligned}
$$

Die Tangente schneidet die x-Achse an der Stelle $x = \frac{3}{5}$.