Aufgabe 1

a)

$$f(x) = 12 \cdot \sin(x)$$

$$f'(x) = 12 \cdot \cos(x)$$

b)

$$f(x) = -2 \cdot \cos(x)$$

$$f'(x) = 2 \cdot \sin(x)$$

c)

$$f(x) = \sqrt{5} \cdot \cos(x)$$

$$f'(x) = -\sqrt{5} \cdot \sin(x)$$

d)

$$f(x) = \frac{1}{\pi} \cdot \sin(x)$$

$$f'(x) = \frac{1}{\pi} \cdot \cos(x)$$

e)

$$f(x) = 5x^3 - \sin(x)$$

$$f'(x) = 15x^2 - \cos(x)$$

f)

$$f(x) = 2\cos(x) - \sin(x)$$

$$f'(x) = -2\sin(x) - \cos(x)$$

Aufgabe 2

$$x_0 = \pi$$

a)

$$f(x) = -9\sin(x)$$

$$f'(x) = -9\cos(x)$$
$$m_t = f'(\pi) = -9\cos(\pi) = 9$$

b)

$$f(x) = 5 + \cos(x)$$

$$f'(x) = -\sin(x)$$
$$m_t = f'(\pi) = -\sin(\pi) = 0$$

c)

$$f(x) = 5x - \cos(x)$$

$$f'(x) = 5 + \sin(x)$$
$$m_t = f'(\pi) = 5 + \sin(\pi) = 5$$

Aufgabe 3

a)

$$f(x) = \cos(x)$$
$$f\left(\frac{7}{4}\pi\right) = \cos\left(\frac{7}{4}\pi\right) = \frac{\sqrt{2}}{2}$$
$$\left(\frac{7}{4}\pi \,\middle|\, \frac{\sqrt{2}}{2}\right)$$

$$t(x) = m_t \cdot x + b$$

$$m_t = f'(x) = -\sin(x)$$
$$f'\left(\frac{7}{4}\pi\right) = -\sin\left(\frac{7}{4}\pi\right) = \frac{\sqrt{2}}{2}$$

$$t(x) = \frac{\sqrt{2}}{2}x + b$$
$$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{7}{4}\pi + b$$
$$\frac{\sqrt{2}}{2} = \frac{7\pi\sqrt{2}}{8} + b$$
$$\frac{4 - 7\pi}{4\sqrt{2}} = b$$
$$t(x) = \frac{\sqrt{2}}{2}x + \frac{4 - 7\pi}{4\sqrt{2}}$$

b)

$$f(x) = 3\sin(x)$$
$$f\left(\frac{5\pi}{3}\right) = 3\sin\left(\frac{5\pi}{3}\right) = -\frac{3\sqrt{3}}{2}$$
$$\left(\frac{5\pi}{3} \,\middle|\, -\frac{3\sqrt{3}}{2}\right)$$

$$t(x) = m_t \cdot x + b$$

$$m_t = f'(x) = 3\cos(x)$$
$$f'\left(\frac{5\pi}{3}\right) = 3\cos\left(\frac{5\pi}{3}\right) = \frac{3}{2}$$

$$t(x) = \frac{3}{2}x + b$$
$$-\frac{3\sqrt{3}}{2} = \frac{3}{2} \cdot \frac{5\pi}{3} + b$$
$$-\frac{3\sqrt{3}}{2} = \frac{5\pi}{2} + b$$
$$-\frac{3\sqrt{3}+5\pi}{2} = b$$

$$t(x) = \frac{3}{2}x - \frac{3\sqrt{3}+5\pi}{2}$$

c)

$$f(x) = x + 2\sin(x)$$
$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + 2\sin\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + \sqrt{2}$$
$$\left(\frac{\pi}{4} \,\middle|\, \frac{\pi}{4} + \sqrt{2}\right)$$

$$t(x) = m_t \cdot x + b$$

$$m_t = f'(x) = 1 + 2\cos(x)$$
$$f'\left(\frac{\pi}{4}\right) = 1 + \cos\left(\frac{\pi}{4}\right) = 1 + \sqrt{2}$$

$$t(x) = (1 + \sqrt{2}) \cdot x + b$$
$$\frac{\pi}{4} + \sqrt{2} = (1 + \sqrt{2}) \cdot \frac{\pi}{4} + b$$
$$\frac{\pi}{4} + \sqrt{2} = \frac{\pi}{4} + \frac{\pi}{4} \cdot \sqrt{2} + b$$
$$\sqrt{2} = \frac{\pi}{4} \cdot \sqrt{2} + b$$
$$\sqrt{2} - \frac{\pi}{4} \cdot \sqrt{2} = b$$

$$t(x) = (1 + \sqrt{2}) \cdot x + \sqrt{2} - \frac{\pi}{4} \cdot \sqrt{2}$$