Aufgabe 21

$$f(x) = \frac{1}{4}x^2 + cx \qquad c \in \mathbb{R}^{\neq 0}$$

$$t(x) = m \cdot x + b \qquad f'(x) = \frac{2}{4}x + c = \frac{1}{2}x + c$$

$$\begin{aligned} f'(0) &= \frac{1}{2} \cdot 0 + c = c = m_1 \\ t_1(x) &= c \cdot x + b_1 \\ 0 &= c \cdot 0 + b_1 \\ 0 &= b_1 \\ t_1(x) &= cx \end{aligned}$$

$$\begin{aligned} f'(a) &= \frac{1}{2} \cdot a + c = \frac{a}{2} + c = m_2 \\ t_2(x) &= \left(\frac{a}{2} + c\right) \cdot x + b_2 \\ 0 &= \left(\frac{a}{2} + c\right) \cdot a + b_2 \\ 0 &= \frac{a}{2} \cdot a + ca + b_2 \\ 0 &= \frac{a^2}{2} + ca + b_2 \\ -\frac{a^2}{2} - ca &= b_2 \\ t_2(x) &= \left(\frac{a}{2} + c\right) \cdot x - \frac{a^2}{2} - ca \end{aligned}$$

Schnittpunkt B der Tangenten:

$$
\begin{aligned}
t_1(x) &= t_2(x) \\
cx &= \left(\frac{a}{2} + c\right) \cdot x - \frac{a^2}{2} - ca \\
cx - \left(\frac{a}{2} + c\right) \cdot x &= -\frac{a^2}{2} - ca \\
x \cdot \left(c - \frac{a}{2} + c\right) &= -\frac{a^2}{2} - ca \\
x \cdot \frac{a}{2} &= -\frac{a^2}{2} - ca \\
x &= \left(-\frac{a^2}{2} - ca\right) \cdot \frac{a}{2} \\
x &= -\frac{a^3}{4} - \frac{ca^2}{2}
\end{aligned}
$$

$$
\begin{aligned}
x &= -\frac{a^3}{4} - \frac{2ca^2}{4} \\
x &= \frac{-a^3 - 2ca^2}{4}
\end{aligned}
$$

$$
t_1\left(\frac{-a^3 - 2ca^2}{4}\right) = \frac{-a^3 - 2ca^2}{4} \cdot c = \frac{-a^3c - 2c^2a^2}{4}
$$