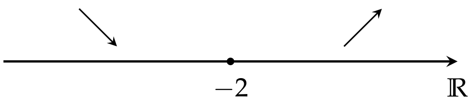
Aufgabe 3

a)

$$\begin{aligned}
f(x) &= \frac{1}{8}x^4 + 4x \\
f'(x) &= \frac{1}{2}x^3 + 4 \\
\frac{1}{2}x^3 + 4 &= 0 \\
\frac{1}{2}x^3 &= -4 \\
x^3 &= -8 \\
x &= -2
\end{aligned}$$

Punktprobe der Steigung:

$$\begin{aligned}
f'(-3) &= \frac{1}{2} \cdot (-3)^3 + 4 = -\frac{19}{2} < 0 \\
f'(-1) &= \frac{1}{2} \cdot (-1)^3 + 4 = \frac{7}{2} > 0
\end{aligned}$$



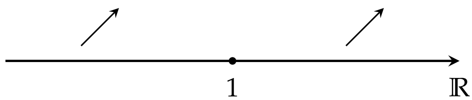
Der Graph von f ist streng monoton fallend im Intervall $(-\infty; -2]$ und streng monoton steigend im Intervall $[-2; +\infty)$.

b)

$$\begin{aligned}
f(x) &= x^3 - 3x^2 + 3x \\
f'(x) &= 3x^2 - 6x + 3 \\
3x^2 - 6x + 3 &= 0 \\
x^2 - 2x + 1 &= 0 \\
x_{1,2} &= \frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 1} \\
&= 1 \pm \sqrt{0} \\
x &= 1
\end{aligned}$$

Punktprobe der Steigung:

$$\begin{aligned}
f'(0) &= 3 \cdot 0^2 - 6 \cdot 0 + 3 = 3 > 0 \\
f'(2) &= 3 \cdot 2^2 - 6 \cdot 2 + 3 = 3 > 0
\end{aligned}$$



Der Graph von f ist streng monoton steigend im Intervall $(-\infty; +\infty)$.

c)

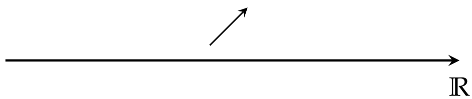
$$\begin{aligned} f(x) &= 4x^3 - 2x^2 + 2x \\ f'(x) &= 12x^2 - 4x + 2 \end{aligned}$$

$$\begin{aligned} 12x^2 - 4x + 2 &= 0 \\ x^2 + \frac{1}{3}x + \frac{1}{6} &= 0 \end{aligned}$$

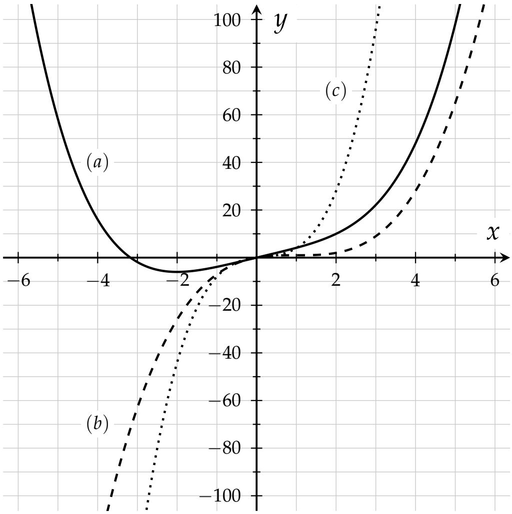
$$\begin{aligned} x_{1,2} &= -\frac{\frac{1}{3}}{2} \pm \sqrt{\left(\frac{\frac{1}{3}}{2}\right)^2 - \frac{1}{6}} \\ &= -\frac{1}{6} \pm \sqrt{\frac{1}{36} - \frac{6}{36}} \\ &= -\frac{1}{6} \pm \sqrt{-\frac{5}{36}} \end{aligned}$$

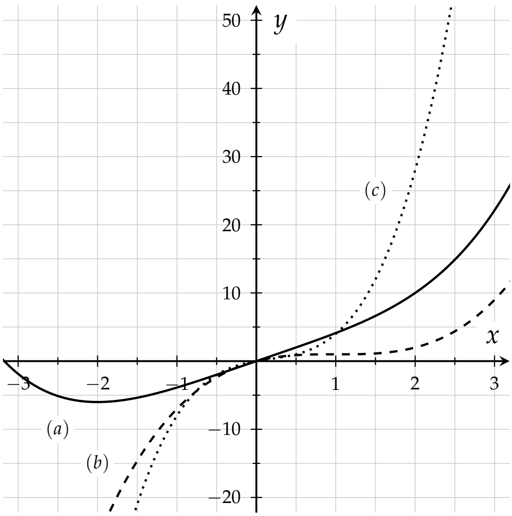
Keine Lösung. Punktprobe der Steigung:

$$f'(0) = 12 \cdot 0^2 - 4 \cdot 0 + 1 = 1 > 0$$



Der Graph von f ist streng monoton steigend im Intervall $(-\infty; +\infty)$.


y
x
100
80
60
40
20
−20
−40
−60
−80
−100
−6
−4
−2
2
4
6
(a)
(b)
(c)


y
x
50
40
30
20
10
−10
−20
−3
−2
−1
1
2
3
(a)
(b)
(c)